**Thinking About Exponents**

**Part 1**

Many adults dream about a day in the future when they can retire from working their job and spend more time on their hobbies, traveling, and spending time with friends and family. To realize this dream, people have to save money so that they can afford to do all the things that they want to do!

Suppose a 35-year-old person was able to save $10,000. They plan to invest this money into a retirement savings account that earns 12% annual interest (by the way, we haven’t seen such a high rate of return since 1989!). This would mean that their money would double in value every 6 years.

1. Based on this investment, complete the table.

|  |  |
| --- | --- |
| Years since investment | Amount of money in the investment |
| 0 |  |
| 6 |  |
| 12 |  |
| 18 |  |
| 24 |  |
| 30 |  |
| 36 |  |

2. Suppose *n* represents the number of 6-year periods that have passed. For example, *n* = 2 means 12 years have passed or *n* = 5 means 30 years have passed. Explain what each of the following expressions mean in the context of this investment situation.

a)

b)

c)

d)

3. Write a formula that could be used to find the amount of money, *A*, in the investment for any value of *n* where *n* still represents the number of 6 year periods that have passed since the initial investment.

4. Use your formula in #3 to determine how much money will be in the investment if the person wants to retire at age 56.

5. Based on the work you have done, write a convincing argument for what you think the value of should be.

6. Suppose a student said that because . What would you tell this student to help them to understand the true value of Explain in detail.

**Part 2**

Suppose you are told that someone invested some unknown amount of money at 12% annual interest so that the amount of money in the account doubled every 6 years. After 30 years, the account had a balance of $850,000.

1. How much did this person invest initially? Explain how you know.

2. Suppose *n* represents the number of 6-year periods that have passed. For example, *n* = 2 means 12 years have passed or *n* = 5 means 30 years have passed. Assume that *n* = 0 corresponds to the current situation where the account balance is $850,000. Explain what each of the following expressions mean in the context of this investment situation.

a)

b)

c)

d)

e)

3. Based on the work you have done, write a convincing argument for what you think the value of should be. What about ? What about ?

4. Write an expression that could be used to compute the amount of money used in the initial investment 30 years ago.

**Part 3**

In this part, there is no context related to money and investments. Rather, you will focus your attention on the mathematical meaning of negative exponents.

1. Complete the table.

|  |  |  |
| --- | --- | --- |
|  |  | 625 |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

2. Describe the pattern you observe and write a convincing argument for why this pattern makes sense.

3. Complete the table.

|  |  |  |
| --- | --- | --- |
|  |  | 625 |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

4. Describe the pattern you observe with the negative exponents and write a convincing argument for why this pattern makes sense.

5. Complete the table. Note that .

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

6. Describe the pattern you observe and write a convincing argument for why this pattern makes sense.

7. Without using a calculator, compute and explain how you know.

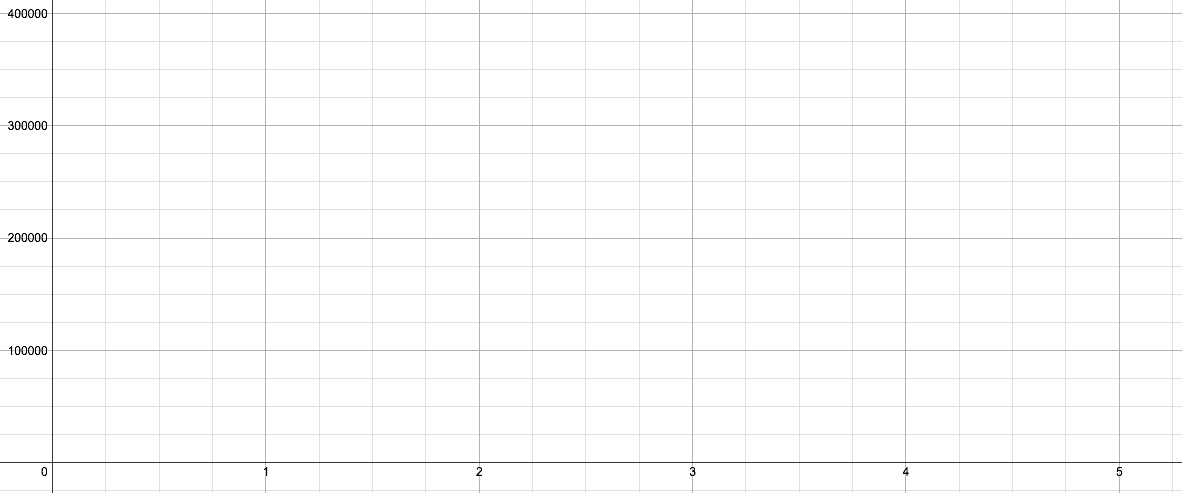
8. Without using a calculator, compute and explain how you know.

9. Explore the idea of . Write a convincing argument for how you think we should think about .

**Part 4 –** Radical and Fractional Exponents

Using the context of saving for retirement, you have had the opportunity to make sense of positive and negative exponents and zero as an exponent. What about fractional exponents like or ?

Let’s revisit the situation from Part 1 where a 35-year-old person was able to save $10,000. They plan to invest this money into a retirement savings account that earns 12% annual interest. This would mean that their money would double in value every 6 years. Suppose *n* represents the number of 6-year periods that have passed. For example, *n* = 2 means 12 years have passed or *n* = 5 means 30 years have passed.

1. In Part 1, you determined a formula for the amount of money, *A*, in the account after *n* 6-year periods is . Create a fairly accurate graph of this function.

2. Explain what each of the following expressions mean in the context of this investment situation and estimate the value of each expression using your graph.

a)

b)

c)

d)

3. Look back at 2c). Someone might claim that the amount of money in the account when *n* = is $15,000 since they expect to see an amount that is half-way between the initial amount ($10,000) and the amount when *n* = 1 ($20,000). Based on the behavior of the graph, explain why this claim is not possible.

4. Examine the table below.

|  |  |  |
| --- | --- | --- |
| Years since investment | *n* | Amount of money in the investment |
| 0 | 0 | $10,000 |
| 3 | 1/2 |  |
| 6 | 1 | $20,000 |

We can correctly say that $10,000 must be multiplied by some factor in order to find the amount of money after 3 years (*n* = ). Then this quantity (whatever it is) must be multiplied by the same factor in order to find the amount of money after 6 years (*n* = 1). That is, when $10,000 is multiplied by some factor twice, the result is $20,000. Write out this explanation using mathematical symbols/equations. Solve the equation and explain what the solution means in the context of this situation.

5. We can write question #4 in mathematical symbols by saying to find out the amount of money in the account after 3 years (*n* = ). Based on your work in #4, what must be the value of ? Explain how you know

6. Using the work you did in #4 and #5, create an argument for the amount of money in the account after 2 years (*n* = )? That is, what is ?

7. In general, what is an equivalent way to express where *c* is an integer such that ?

8. Develop an argument for the following claim. That is, try to create a convincing argument to support the following statement.

For any and for an integer *c* such that , .